MARMARA UNIVERSITY
COMPUTER SCIENCE ENGINEERING
LINEAR ALGEBRA
MIDTERM HOMEWORK
29/05/2020

Write your solutions on a standard white paper. You can use at most 3 papers. You should write your name, last name, student number, the declaration of truth (DOĞRULUK BEYANI: Bu sınav kağıdındaki tüm sorular tarafımca çözülmüş olup, hiçbir yolla başka bir şahıstan yardım alınmamış ve başka bir şahısla paylaşılmamıştır.) and your signature on each paper. Scan your solution papers possibly with your phone and upload it to ues.marmara.edu.tr as a single pdf file with file name name_lastname before 14:00. If you encounter upload problems, you can send your solution pdf file to the email address taylan.sengul@marmara.edu.tr. Each problem is worth 20pts. Show your solution steps clearly to get credit. Good luck!

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1. Are the following true or false? If true, give a reason. If false, give a counterexample.

   1. $\det(I + A) = 1 + \det A$, for every 3×3 matrix A.
   2. $\det(4A) = 4\det(A)$ for every 5×5 matrix A.
   3. For every 2×2 matrices A and C, if $AC = O$ then either $A = O$ or $C = O$ where O is the 2×2 zero matrix.
   4. For every 3×3 matrix A if $\det(A^3) = 0$ then $\det(A) = 0$.

Solution: (1) False. Choose $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Then $\det(A) = 1$, $\det(I + A) = 0$.

(2) False. Choose $A = I_5$. Then $\det(4A) = 4^5$, $\det(A) = 1$.

(3) False. Choose $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

(4) True since $\det(A^3) = \det(A)^3$.

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2. Compute $\det \begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{pmatrix}$, by applying row operations to produce an upper triangular U.

Solution: $-2r_1 + r_2 \to r_2$, $r_1 + r_3 \to r_3$ $B = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 2 & 3 & 3 \\ 0 & 2 & 0 & 7 \end{pmatrix}$

$-r_2 + r_3 \to r_3$, $-r_2 + r_4 \to r_4$ $C = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 6 \end{pmatrix}$

Determinant is $= \det(B) = \det(C) = 1 \cdot 2 \cdot 3 \cdot 6 = 36$.

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3. For $A = \begin{pmatrix} a & 1 & 0 \\ b & 0 & 1 \\ 0 & c & 1 \end{pmatrix}$, find the adjugate matrix $\operatorname{adj}(A)$.

Solution: $\operatorname{adj}(A) = \begin{pmatrix} -c & -1 & 1 \\ -b & a & -a \\ bc & -ac & -b \end{pmatrix}$

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4. Write the matrix $\begin{pmatrix} 1 & 4 \\ 2 & 7 \end{pmatrix}$ as a product of elementary matrices.

Solution: $-2r_1 + r_2 \to r_2$: $\begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & 7 \end{pmatrix}$

$-r_2 \to r_2$: $\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix}$

$-4r_2 + r_1 \to r_1$: $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

$I = \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & 7 \end{pmatrix}$

$\begin{pmatrix} 1 & 4 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

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5. Determine the relation between a, b so that the below system is consistent. Find the solution in that case.

$$\begin{aligned} x + 2y - 3z &= a \\ 2x + 3y + 3z &= 0 \\ 5x + 9y - 6z &= b \end{aligned}$$

Solution: $-2r_1 + r_2 \to r_2$, $-5r_1 + r_3 \to r_3$: $\begin{pmatrix} 1 & 2 & -3 & a \\ 0 & -1 & 9 & -2a \\ 0 & -1 & 9 & b-5a \end{pmatrix}$ $-r_2 + r_3 \to r_3$: $\begin{pmatrix} 1 & 2 & -3 & a \\ 0 & -1 & 9 & -2a \\ 0 & 0 & 0 & b-3a \end{pmatrix}$

The system is consistent if $b = 3a$. In that case, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -15r - 3a \\ 9r + 2a \\ r \end{pmatrix}$